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The Future of Mathematics in Economics: A Philosophically Grounded Proposal

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Abstract The use of mathematics in economics has been widely discussed. The philosophical discussion on what mathematics is remains unsettled on why it can be applied to the study of the real world. We propose to get back to some philosophical conceptions that lead to a language-like role for the mathematical analysis of economic phenomena and present some problems of interest that can be better examined in this light. Category theory provides the appropriate tools for these analytical approach.

Keywords Foundations of mathematics · Universals in singular objects · Mathematical economics · Category theory · Circularity · Data

1 Introduction

The use of mathematics in economics is a debated topic. Numerous voices claim that economics has become excessively mathematical in its formulation. Other, also numerous, voices not only defend but strongly argue for using and refining the mathematical tools applied to economics. The first group, say of "contrarians", sustains that economics, due to its mathematical formalization, has become an abstract discipline detached from reality, or at the very best only able to show an oversimplified picture of facts. The "partisans" maintain, instead, that mathematics provides the right language to express real-world relations and the tools to reason correctly about them. These defenders of the

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mathematically of economics can, in turn, be split in two. On one hand, we have those who support axiomatical presentations of the ideas while on the other, we have those who stress the need of a more empirically based mathematical approach. The former tradition focuses on building logically coherent bodies free from psychological subjective biases while the latter criticizes the speculative character of a priori approaches and support a two-way interaction between empirical data and formal models. While each side has some valid points, it is interesting to note that both the contrarians and the empirically leaning partisans concentrate their attacks on the economists that build purely abstract models. This latter position can be seen as being at the core of the Mathematical economics tradition. Since it has been subservient to specific meta-conceptions of mathematics, their analysis will help to choose the right mathematical approach on which to base any future developments in this area.

In this paper we will discuss these problems along two lines of argumentation. On one hand we will discuss the shortcomings of the traditional foundations of the mathematical tools in economics. On the other, we will introduce new approaches and explore their consequences, discussing the meaningfulness of the results and their future perspectives.

The rest of this paper is organized as follows. In Sect. 2 we will sketch the main traditional philosophical conceptions of mathematics. At the end of this section, after this review, we note that an empiricist aristotelian conception of mathematics has been neglected during the twentieth century. Sections 3 and 4 extend this discussion to the possible application of mathematics in the empirical sciences and to economics in particular. The specific requirements of these types of sciences fit with the characterization of the aristotelian position, which implicitly promotes ways of doing mathematics complementary to those currently used in mathematical economics. Section 5 discusses some of those future lines of development. It shows how some mathematical instruments unsatisfactory for managing some economic features could be successfully replaced by other tools coming from category theory. Section 6 concludes.

2 Philosophical Views on Mathematics

Philosophers have reflected on the nature of mathematical thinking since its inception. But the first predominant conceptions of mathematics were formulated by Plato and Aristotle. Afterwards, they remained without major changes until the 1800s. In this section we will briefly introduce the modern conceptions that arose at the turn of the twentieth century. One of these positions is *platonism*, obviously alluding to Plato, but only vaguely inspired in him. Aristotle's ideas, instead, exerted almost no influence on the modern currents in philosophy of mathematics.

According to Kitcher and Aspray, the German mathematician, logician and philosopher Gottlob Frege "posed the problems with which philosophers of mathematics have struggled ever since" (Kitcher and Aspray 1988, p. 3ff). Frege maintained that the key problem of philosophy of mathematics was to identify its foundations. This idea is still present in different guises. The main ones are the following:

- *Platonism* In truth, platonism in mathematics, as mentioned, is only loosely inspired in Plato's ideas. The label is inspired in Plato's thought on the existence of ideas independent of the empirical objects of which they are as the archetype. More precisely, mathematical platonism has been defined as the conjunction of three theses. Namely, *existence* (there are mathematical objects), *abstractness* (mathematical objects are abstract) and *independence* (mathematical objects are independent of intelligent agents and their language, thought, and practices) (Linnebo 2013). Thus, mathematical entities, according to platonists, are really existent beings and are, instead of physical, abstract objects. The truth of mathematical statements derives from the objects to which they refer: the truth is discovered, not constructed. While one of its fathers was Frege, many (and maybe most) mathematicians share it. In one way or other, either supporting or criticizing it, this conception crosses all the different visions of mathematics. But it still has to answer a relevant question of epistemological and ontological nature: how do we have access to mathematical objects (see Benacerraf 1973)?

- *Logicism* Another strand in the philosophy of mathematics initiated by Frege is based on the idea that mathematical concepts are defined in logical terms, derived from logical laws and, in sum, reduced to logic (see Carnap 1964, p. 31): "Arithmetic would be only a further developed logic, every arithmetic theorem a logical law, albeit a derived one" (Frege 1964, p. 107). This program was continued by Bertrand Russell and Alfred Whitehead in **Principia Mathematica**. The very nature of the logicist approach is summarized by the following claim: "Philosophy asks of mathematics: What does it mean? Mathematics in the past was unable to answer, and philosophy answered by introducing the totally irrelevant notion of mind. But now mathematics is able to answer, so far at least as to reduce the whole of its propositions to certain fundamental notions of logic" (Russell 2010, pp. 3–4, Chapter I. Definition of Pure mathematics). This reveals that in a certain way, Russell was a platonist realist; for him, as for his contemporary colleague George E. Moore, logical entities are real objects. Other logicist positions admitted, instead, a conventional foundation. Many criticisms of logicism have been advanced, based on the idea that not all mathematics can be reduced to logic (Benacerraf and Putnam 1964, p. 10). Nevertheless, the deepest criticisms go against its platonism. More precisely, since logic deals with signs -second intentions, which are not more than signs-, reducing mathematics to logic either implies a platonist position (thinking that these second intentions are real abstract objects) or a non-realist stance (if second intentions are seen as having a lower ontological status). As noted by W.V.O. Quine, old medieval claims on the nature of "universals" reappeared in the twentieth century at the root of these different views. So, for instance, the platonist and the logicist camps conceives universals as abstract real objects (Quine 1964, p. 192).
- *Intuitionism* This foundational program was initially developed by the Dutch mathematician Luitzen E. J. Brouwer, for whom "man always and everywhere creates order in nature" (Brouwer 1999, p. 57). The idea is that humans construct the objects of mathematics and that our knowledge of their fundamental properties is based on an a priori intuition. Through this intuition, we are able to recognize a potential infinity of mathematical entities (the natural numbers), and these can form the basis for further constructions (Kitcher and Aspray 1988, p. 7). In this way, "[t]he truth of a mathematical statement can only be conceived via a mental construction that proves it to be true" (Iemhoff 2013, p. 1; see also Heyting 1964, p. 42). Mathematics, for intuitionists, consists in building mental constructions which are "inductive and effective". Furthermore, in this view classical procedures cannot be used, like proving claims by contradiction or applying the *axiom of choice*. But this leads to the rejection of some universally accepted theorems and, in consequence "the mathematical community has almost universally rejected intuitionism" (Snapper 1979, p. 211). With respect to the intuitionist position on the nature of mathematical objects, we have that "Intuitionists affirm the existence of mathematical objects but maintain that these

objects depend on or are constituted by mathematicians and their activities" (Linnebo 2013, p. 5). In other words, they are conceived as being mental objects. This fits well with the medieval conception of universals called *conceptualism*, which "holds that there are universals but they are mind-made" (Quine 1964, p. 192). Mathematical objects are thus invented, not discovered. Or as Brouwer claims, mathematical exactness exists in the human intellect (Brouwer 1999, p. 56).

- *Formalism* A fourth foundational program developed in the twentieth century was started by David Hilbert, who advocated for the full axiomatization of mathematical theories in order to ensure that they are free of contradictions. In this view the most important task of mathematics is to provide consistency proofs of mathematical theories. On the other hand, the meaning of the formal constructions is irrelevant: "for the strict formalist to do mathematics is to manipulate the meaningless symbols of a first order language according to explicit, syntactic rules" (Snapper 1979, p. 50). But in 1931 Kurt Gödel proved that mathematical theories constructed under this approach are either incomplete or cannot be proven to be free of contradictions. However, it has been clearly shown that Hilbert conceived formalism only in relation to the foundations instead of the whole body of mathematics (Corry 1997, 2002, 2004). Other mathematicians took the extreme position of applying formalism to the whole corpus of mathematics. This is the case of the group of French mathematicians known under the pseudonym of Nicolas Bourbaki. Mathematics, for Jean Dieudonné, one of the bourbakians, is not more than a play of chess: "On foundations we believe in the reality of mathematics, but of course when philosophers attack us with their paradoxes, we run to hide behind formalism and say: 'mathematics is just a combination of meaningless symbols'." (cited in Corry 2004, p. 310). It is fair to say that Hilbert did not go as far as the extreme bourbakian position, being also concerned with the application of mathematics to the real world (Weintraub 2002 Chapter 3; Giocoli 2003, pp. 18–20; Giocoli 2009. pp. 7–9). However, the general flavor and emphasis of Hilbert's ideas was on axiomatic consistency. While for mathematicians of the nineteenth century (for instance Vito Volterra) mathematical rigor meant correspondence to real facts, for Hilbert it was a "consistent outcome of a deductive process applied to the assumptions, rather than based upon the observation of reality" (Giocoli 2003, p. 23). In effect, at the turn of the twentieth century the meaning of rigor in mathematics shifted from empirical correspondence to axiomatic consistency (Weintraub 2002, pp. 17, 34, 48–51, 22–23; Giocoli 2003, p. 5). With respect to the nature of universals in formalism, Quine maintains that it holds the nominalist position (Quine 1964, p. 193), that is, universals are no more than *flatus vocis*, empty or mere names, words, or sounds without corresponding objective realities. Curiously enough, this is the source on which Hilbert based his belief in the power of mathematics as an unifying cornerstone for all scientific endeavors (Giocoli 2003, p. 24).

For different reasons, platonism, logicism, intuitionism and formalism have failed to provide mathematics with the philosophical foundations Frege looked for. Modern mathematics has in fact abandoned this quest despite that some authors think that the philosophy of mathematics holds the key to a firm foundation for the discipline (Snapper 1979, p. 216). In this sense, it is interesting to note that from the medieval views on universals, one has not been adopted by the aforementioned alternative foundations of mathematics. It is the one based on Aristotle's idea that universals are present in singular concrete objects as their essence (*universal in essendo*), in concepts expressing this essence (*universal in repraesentando*), and in words referring to the plurality of things that share the same feature (*universal in significando*). Thus, while platonic realism refers to abstract objects, conceptualism to objects only existing in the mind and nominalism denies the existence of universal categories, the aristotelian view refers to real world things. The latter position has been neglected in the discussion on the foundations of mathematics. More precisely, there has been no empiricist tradition in the philosophy of mathematics in the early twentieth century (Kitcher and Aspray 1988, p. 8). But then, this could have been the right foundations for a mathematics of the empirical sciences like economics.

3 What Mathematics for the Real World?

A reasonable take on this matter should be based on the idea that mathematics (be it formalism's consistency, intuitionism's insights or logicism's laws) should be in some correspondence with real facts. To verify this, empirical tests may be needed. Interestingly enough, as noted by Eugene Wigner, such tests may have been passed successfully: "mathematical language has more to commend it than being the only language which we can speak; it shows that it is, in a very real sense, the correct language" (Wigner 1960). But, why? He leaves us without explanations (Wigner 1960, p. 216):

> The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning.

Kline even talks about a "magical power" and an "inner mysterious strength" (Kline 1985, p. 216). There is a huge literature trying to solve this enigma. But the explanation should come from a relation between mathematics and the actual world. The formalist, the logicist nor the intuitionist lines of thought have an answer to this question. But, as already mentioned, Aristotelian realism addresses this point by explicitly locating the foundations of mathematics in the relation between their concepts and empirical reality. This view has been recently rehabilitated by the School of Sidney and other mathematicians who think that mathematical objects are "fictional entities grounded on physical objects" (Mendell 2004, p. 2). James Franklin, for instance, sees mathematics as the study of quantitative and structural aspects of things, like ratios, patterns, or relations: "[mathematicians] are scientists who study patterns or forms that arise in nature" (Franklin 2009, p. 103). The idea is that mathematical symbols are "instrumental" signs of mathematical concepts which are either "formal" or "natural" signs of real structures or properties (Franklin 2009, pp. 101–102):

> Aristotelian realism unifies mathematics and the other natural sciences. It explains in a straightforward way how babies come to mathematical knowledge through perceiving regularities, how mathematical universals like ratios, symmetries and continuities can be real and perceivable properties of physical and other objects, how new applied mathematical sciences like operations research and chaos theory have expanded the range of what mathematics studies, and how experimental evidence in mathematics leads to new knowledge.

In this sense, a mathematical proposition would just express a well-founded empirical generalization or law about the properties and behavior of objects (Gasking 1964, p. 391).

As Kline (Kline 1985, p. 203) asserts, "there is something in the external world that mathematical theory can capture and encapsulate". The consistency of mathematical developments might express the rationality of the real world. As Pierre Duhem concludes, "it is impossible for us to believe that this order and this organization produced by theory are not the reflected image of a real order and organization" (quoted in Kline 1985, p. 220). Consequently, though we can imagine mathematical developments without previous induction, they could reflect a (maybe yet unknown) piece of reality. That is, mathematics would then be the science of possible consistent objects.

In concrete applications it becomes fundamental to check whether these constructions can be really found in the world. When we want to apply these developments to the analysis of natural or social sciences we are concerned with actually real properties. This is why we have to check whether these constructions are really found in the world. For Aristotle, the priority belongs to reality over theory. He says in Generation of Animals (concerning his observations about the generation of bees) that "credit must be given rather to observation than to theories, and to theories only if what they affirm agrees with the observed facts" (III 10, 760b 31; cf. also De Anima, I, I, 639b 3 ff. and 640a 14 ff.). This position parallels a realist view of measurement which considers that the latter is based on quantitative attributes of reality (see for example Michell 2005).

However, we must consider that it is usually easier to use data to check out a theory in the natural than in social sciences. In the latter there is a kind of complexity, aggravated by human freedom, that makes patterns irregular and predictions very difficult. The usual way of testing the validity of claims, by resorting to data provided by experiments or field surveys shows this complexity. For instance, psychology (which in large parts overlaps with behavioral economics) is plagued with reproducibility problems, making the evidence provided by experiments and surveys of little help for the evaluation of extremely precise claims.[1] Predicting real world behaviors is thus a very involved activity and can be achieved, as MacIntyre explains applying Aristotle's ideas, only if (a) statistical regularities are known; (b) there are systematical ways in which people schedule and coordinate their social actions; and (c) there are known causal regularities between natural and social life (MacIntyre 1984, pp. 102–103).

Aristotle considers these difficulties involved in social sciences. For him, scientific rigor means adaptation to empirical facts, and thus, given the idiosyncratic nature of human facts, social theories cannot be universal. Aristotle asserts in his Nicomachean Ethics referring to politics which is for him the most "architectonic" human science (I 3 1094b 13–14 and I 7 1098a 28–29):

> Now our treatment of this science will be adequate, if it achieves that amount of precision which belongs to its subject matter. The same exactness must not be expected in all departments of philosophy alike, any more than in all the products of arts and crafts (…) We must therefore be content if, in dealing with subjects and starting from premises thus uncertain, we succeed in presenting a broad outline of the truth: when our subjects and our premises are merely generalities, it is enough if we arrive at generally valid conclusions.

Science should not be demanded more than it can say in relation with the nature of its subject. This limitation is not shameful, since it does not originate from a weakness of science but, as Aristotle also says, from "the nature of the case: the material of conduct is

[1] The pervasiveness of this problem prompted a large initiative to address it, leaded by Brian Nosek (Baker 2015).

essentially irregular" (Nicomachean Ethics, V, 10, 1137b 17-9). As Richard Kraut asserts, Aristotle "is asking us to have different expectations of different fields: not *higher* standards for some fields and *lower* for others, but *different* standards" (Kraut 2006, p. 87). Furthermore, in many cases the knowledge is sought to solve practical problems. In this case, the rigor required in their solution is quite different from the rigor necessary in axiomatic systems: "Practical issues require a different set of rules than axiomatic problems. This means that the rigor applied in solving practical problems will inevitably be different to the rigor in an axiomatic system" (Boumans 2005, p. 151). More precisely, the range of validity of conclusions must be contextualized, yielding only *local* knowledge.

We can summarize saying that the validity of a social or economic knowledge is grounded in local theories confirmed by data. Any generalization of local theories requires a context that may involve natural laws, institutions, social routines or structures. The mathematical tools needed to build such theories should be able to somehow express the fluctuating and inexact nature of social reality. This matches well with Aristotle's philosophy of mathematics in which its main role is to capture, the best it can, the regularities of the real world.

4 From Debreu's to von Neumann's View

Authors like Weintraub and Giocoli have examined the history of the relations between mathematics and economics in the last century, finding that the abstract character of the different currents in the philosophy of mathematics -platonism, formalism, logicism, and intuitionism- had a great influence on the way in which economic theories were built. These authors considered two different kinds of questions in the discipline. The first kind involves its *body of knowledge*, i.e. the theories, facts and methods used in economics. The other questions refer to the *image of knowledge*: the problems that deserve to be tackled in economics and the perspectives adopted. Thus the increasing demand for mathematical rigor in economics would be a result of the change of image of the discipline, from a mere instrument for other disciplines to a self-containing full-fledged science. A related change of image led economics from being conceived as the study of *systems of forces* to be seen as the examination of *systems of relations* (Giocoli 2003, pp. 4–6 and *passim*):

> According to system-of-forces (SOF) view, economics is a discipline whose main subject is the analysis of the economic processes generated by market and non-market forces, including—but by no means exclusively—the processes leading the system to an equilibrium. According instead to the system-of-relations (SOR) view, economics is a discipline whose main subject is the investigation of the existence and properties of economic equilibria in terms of the validation and mutual consistency of given formal conditions, but which has little if anything to say about the meaningfulness of these equilibria for the analysis of real economic systems.

There is a parallelism between this shift of image and mathematics' shift of image, including the concept of rigor. This shift does not mean that the interest on economic applications has been lost, but that the formal tendency has prevailed. As Giocoli asserts, "what really drove the transformation of modern neoclassical economics in the direction of formalism and mathematics was the economists' desire to achieve the greatest possible generality and conceptual integrity of their analysis" (Giocoli 2003, pp. 15). This scientific goal was naturally encouraged by the increase in rigor of mathematical formalisms. Thus, according to this author, economics became a branch of logic "concealed behind an empiricist façade" (Giocoli 2009, p. 23).

The formalist tendency is clear in Gerard Debreu's case. His connections with the Bourbaki group, the circumstances of his incorporation to the Cowles Commission, and his contacts with the American bourbakian mathematicians have been widely discussed (Weintraub 2002, Chapter 4; Giocoli 2003, pp. 117–124; Düppe 2011, pp. 166–167). In *The Mathematization of Economic Theory*, Debreu asserts: "Being denied a sufficiently secure experimental base, economic theory has to adhere to the rules of logical structure and must renounce the facility of internal inconsistency" (Debreu 1991, p. 2). He also states that "in proving existence one is not trying to make a statement about the real world, one is trying to evaluate the model" (quoted in Düppe 2011, p. 168). It goes without saying that Debreu's work had an enormous impact on Economic Theory. Samuelson even speaks about an "Age of Debreu" in the discipline (Samuelson 1998, pp. 1375–1386).

It is interesting to contrast the case of Debreu to John von Neumann's. The latter, a former assistant of David Hilbert,[2] held in economics a view closer to the system-of-forces image than to the system-of-relations one (Giocoli 2003, pp. 212–214). von Neumann and Oskar Morgenstern's *The Theory of Games and Economic Behavior* (1944) showcased this view: "what von Neumann/Morgenstern had done, in fact, was to devise a notion of rationality that combined formal rigor with the possibility of accounting for the 'how and why' of the agents' behavior" (Giocoli 2003, p. 10).

Despite his formalist origins, von Neumann was aware of the need of empirical sources in the applications of mathematics (von Neumann 1960, p. 2063):

> As a mathematical discipline travels far from its empirical source, or still more, if it is a second and third generation only indirectly inspired by ideas coming from 'reality', it is beset with grave dangers [...] In other words, at a great distance from its empirical source, or after 'much 'abstract' inbreeding, a mathematical subject is in danger of degeneration.

For von Neumann, the solution was clear: "Whenever this stage is reached, the only remedy seems to me to be the rejuvenating return to the source: the reinjection of more or less directly empirical ideas" (von Neumann 1960). According to Giocoli, economics was not ready to adopt these ideas and this explains why von Neumann did not further work related to this discipline.

Another interesting case to consider is that of Wassily Leontief, who was a fervent advocate of the use mathematical models in conjunction with empirical data. In his presidential address to the American Economic Association (1970), he referred to the lack of empirical backing for many theories, stating, for example, that "[t]he weak and all too slowly growing empirical foundation clearly cannot support the proliferating superstructure of pure or, should I say, speculative economic theory" (Leontief 1971, p. 1). Indeed, between 1970 and 1980, for example, more than half of the articles published in the American Economic Review used mathematical models without data (Fox 1997, p. 9). This tendency has since been progressively reverted.[3] It is true that it is difficult to obtain reliable, homogenous and repeatable data to undertake real experiments. But data has been collected relentlessly in recent years. Still, in the presentation of the hundredth volume of

[2] About their close connection, see Yasugi and Passell (2003).

[3] "Economic theory may have become so abstruse that editors of the leading general journals, recognizing that very few of their readers could comprehend the theory, have cut back on publishing work of this type" (Hamermesh 2013, p. 162).

the *Journal of Econometrics* (2001), Nobel prize-winning economist James Heckman points to the persistence of a threefold division: Economic Theory (including Game Theory), Mathematical Statistics (favored by theoretical econometricians) and the collection of statistical observations. The future of empirical analysis depends on the combination of these three elements—without one of the three, it becomes useless (Heckman 2001, p. 5). Impediments to this convergence stem from the sophistication of these branches, each one with its own approaches and communities. But on the other hand, there are interesting calls for the unification of these disciplines,[4] along the lines promoted in the following cite (Backhouse 1998, pp. 1856–1857):

> What is the solution? It is emphatically not to abandon formal techniques altogether—that would be to court disaster. What is required is, as von Neumann points out, that theory be kept close to its empirical roots. This in turn requires: (1) that economists put sufficient effort into empirical work [. . .]; (2) that they take empirical evidence seriously; (3) that they be aware of the conceptual gap between theories and reality [. . .]; and (4) that they be aware that mathematical imperatives may result in changes in the questions that are being addressed.

Even without intending to do so, von Neumann, Leontief and Backhouse are proposing a way of doing mathematics according to the aristotelian philosophical conception. In effect, they are promoting a mathematical formalization of economic phenomena in which the empirical aspects have primacy. In this way, the mathematical constructions proposed will be really grounded in the world. The next section will present new mathematical visions and devices that we think can contribute to achieve these goals.

5 The Future

The previous section presented some of the more influential views the role of mathematics in economics put in the context of the foundations of mathematics, personified by the positions of Debreu and von Neumann. But it is interesting to note that more recent advances may lead to further reconsideration. The developments of the last few decades have shown that there are alternative and more convenient ways of representing mathematical ideas. Central to these advances has been the role of *category theory*. There exists a wide consensus among pure and applied mathematicians that "category theory has come to occupy a central position in contemporary mathematics and theoretical computer science, and is also applied to mathematical physics . . .Category theory is an alternative to set theory as a foundation for mathematics." (Marquis 2015, Introduction).

Unlike the traditional foundations, which are much more concerned with the nature of mathematical objects, category theory focuses on the *relations* among them. Intuitively, while set-theoretical foundations define functions in terms of their domain and range sets, category theory takes functions by themselves as the elements of interest. More precisely, any category can be described by the *morphisms* between its *objects*. While this could seem as a mere rewriting of the traditional form of doing mathematics, its real strength arises when different categories are connected trough *functors*, that act both over the objects and

[4] The Santa Fe Institute (http://www.santafe.edu/) is actively engaged in this enterprise. Some of its members have produced interesting manifestos, describing some proposals for the unification of the economic and social disciplines, either by rethinking the notion of *rationality* (Gintis 2009) or by using models of complex systems for their description (Brian Arthur 2015).

the morphisms between them. Moreover, this way of relating mathematical constructions can be pursued further, establishing other kinds of relations.[5] The decline of Bourbaki's approach can be seen as a consequence of the success of category theory: "Switching to categories [in the Bourbaki framework] would mean revising the whole body of previous work, while combining the two approaches seem to undermine the bourbakist ideal of unity of mathematics" (Beaulieu 1999, p. 236). But, "ironically, the bourbakist vision of unified mathematics found a powerful vehicle in category theory. General category-theoretical concepts have enjoyed wide use across many different mathematical fields; in fact the very rigid division of mathematics into fields have been called into question" (Etingof et al. 2011, p. 198).

While this process of "categorification" of mathematics has been going on for many decades, some individual authors like (Lawvere and Schanuel 2009; Goldblatt 1984; Lambek 1994; Caramello 2010), among many, have advocated for an explicit program of unification of mathematics by means of the category-theoretical language. A categorical structure of particular interest in this respect is the *elementary topos*, which "[...] is a category possessing a logical structure sufficiently rich to develop most of 'ordinary mathematics', that is, most of what is taught to mathematics undergraduates. As such, an elementary topos can be thought of as a categorical theory of sets" (Marquis 2015, Section 2). Furthermore, Lambek has even proposed the *free topos*[6] as the suitable candidate for the world of mathematics, noting that the free topos is an object of itself (Lambek 2004, p. 153). A related program of unification of mathematics under a categorical umbrella is *homotopy type theory*, which intends to provide tools for computer-checking mathematical proofs, based on the use type theory to express the proofs, with an interpretation grounded in *homotopy theory*.[7] Since homotopies can be defined among homotopies, and homotopies among homotopies among homotopies, etc., this semantics can be seen as grounded in higher order categories (i.e. with morphisms among morphisms, morphisms among morphisms among morphisms, etc.) (The Univalent Foundations Program 2013, Chapter 2).

The conceptual advantages for economics of basing its formalisms on a categorical setting are manifold. On one hand, category theory changes the focus from *objects* to *morphisms*. This is a move that could be interpreted as favoring the SOR approach, but contrary to Debreu and his bourbakian stance, the categorical focus on morphisms frees the economic models of the emphasis on *equilibria*, which become objects that may or may not exist in the appropriate category. Instead, the relational aspect of morphisms allows to capture a variety of approaches in a single framework. Unlike the bourbakian approach, based on ZFC (Zermelo-Fraenkel's set theory with the axiom of choice), which demands that every entity must be defined in terms of simpler entities, category theory favors a "synthetic" approach, in which objects are given without any consideration to their inner

[5] See Lawvere and Schanuel (2009) of Spivak (2014) for very clear and intuitive yet rigorous introductions to categories.

[6] A model of 'ordinary mathematics' developed in the framework of *type theory*. The latter, introduced by Bertrand Russell to avoid contradictions in naive set theory (Russell 2010) has become a fundamental tool in theoretical computer science, since it allows a pure symbolic manipulation by means of clear rules of how different kinds (*types*) of symbols (*terms*) interact: "It thus emerges that computational type theory is a plausible foundation for computer science as well as for computational mathematics" (Constable 2009, A foundation for computer science).

[7] An homotopy formalizes the intuitive idea of deformability of one mapping into another.

structure, just by its interactions with other objects.[8] This applies both to the axiomatization approach a la Debreu and to the more contemporary approach of "model-building" in game theory, implementation theory, macroeconomics, etc. In the former case, category theory captures the relations among the mathematical universes corresponding to different axiom systems. On the other hand, the model-building approach, grounded in practices more common among computer scientists and physicists, can be clarified by means of categorical tools: "A major tool is the structure of information itself: how data is made meaningful by its connections both internal and outreaching to other data [...] Giant databases are currently being mined for unknown patterns [...] Similarly, in science there exists substantial expertise making brilliant connections between concepts" (Spivak 2014, p. 6).

This a radical departure from the typical approaches to the mathematization of economics. It assumes a web of relations in the real world, ready for the economist to capture the entities and concepts of interest. This embodies the aristotelian conception in a very natural way: assuming that "reality" is captured in data (even in the form of accepted pieces of knowledge), the main mathematical activity of economists is to establish connections among them. This yields the extra bonus of providing the most natural choice for the real-world interpretation of economics, becoming a straightforward framework for data analysis.

Even if in economics this approach has still to gain momentum, it has a history of at least five decades, responding to the need of better notations, allowing more abstract and detached (from any concrete details) characterizations of mathematical objects and relations. Starting with Grothendieck's successful reconstruction of algebraic geometry (Grothendieck 1986) and followed by the advances in the *Langland's program* (Frenkel 2005), the trend towards the unification through abstraction of large swats of mathematics has made the usefulness of a flexible yet rigorous language more evident than ever to mathematicians. Without delving much into Wigner's question, it has also become clear in the sciences that mathematical tools of representation and analysis are necessary at the very least as a language ensuring precision and clarity (Spivak 2014, p. 24ff). Furthermore, given the increasing influence of computer science models in the representation of real world phenomena, the use of category theory in the expression of scientific theories has only grown in the last decades. Economics, as said, has been slow in adopting this new framework. This is even more surprising considering the widespread influence of the claim of Paul Samuelson that mathematics is for economics just a *language* (Samuelson 1952, p. 56). If so, at the very least for its pure notational value, category theory should have been more widely applied in the discipline.

In any case, this is relevant when we consider the slow drift from economic models based on linear algebra and functional analysis to the more abstract tools used in, for instance, the study of mechanism design and epistemic game theory.[9] Some of the core aspects in these investigations can be better seen in the light of categorical representations:

- The understanding of back and forth relation between **individual** and **aggregate** behavior.

[8] See the thorough discussion of the differences between the axiomatic method proper of ZFC and category theory in Rodin (2014).

[9] See Pacuit and Roy (2015) for a description of the latter field. It can be seen that its mathematical framework departs from the classical one described as: "The current period [1960 to the late 1970s] is one of integration, in which modern mathematical economics combines elements of calculus, set theory and linear models" (Arrow 1981, p. 6).

- The need for, on one hand, the a priori modeling process and, on the other, the examination of **real world data**, with the hope that these two approaches will, in the end, converge.
- The *context* of decision-making which should, in turn, *emerge* from the decisions made.

There are many interesting consequences of addressing these problems. On the theoretical side these concerns make economics closer to some areas of computer science and physics. In fact, a large body of research in the former discipline involve transactions in distributed virtual "societies" (Shoham and Leyton-Brown 2009). In the latter, the role of exchanges in quantum contexts are being avidly studied, noting its similarities with intentional settings (Khrennikov 2010).

On the practical side, the aforementioned studies open the door to the design of better tools for decision-making and economic interactions. Particularly important is the possibility of profiting from the abundance of information (big data) to make inferences in complex and noisy environments.

Let us discuss these and related issues in turn.

5.1 Circularity

Economics has always recognized the important role of beliefs, expectations, assumptions, etc. in decision-making. In particular, when these mental constructs refer to the behavior of other agents. But maybe since Keynes' introduced the "beauty contest" analogy[10] it is clear that the main issue of analysis must be the beliefs about the others' beliefs. That is, any agent analyzing a situation has to figure what the other agents think, including what the others think about his beliefs. A "solution" has, by definition, a circular nature, since the beliefs of an agent depend transitively on his own beliefs (mediated through the beliefs of other agents).

This circularity has been shown hard to tame. Very involved arguments, mostly based on theorems on stochastic processes over different spaces, have ensured the existence of fixed point in the process (Mertens and Zamir 1984; Brandenburger and Dekel 1993). But only with the aid of category-theoretical tools more general results have been obtained, less dependent on the structure of the space of states of nature (i.e. non-intentional aspects affecting results) (Vassilakis 1991).

An analogous problem appears in the foundations of the mechanism design literature. If the choice of rules is what matters, then under which rules should this decision be made? And under which ones is *this* decision made, etc., etc. That is, the goal is to find a fixed point in this process of rules-changing rules of changing rules... (Lipman 1991; Vassilakis 1991). Again, tools drawn from category theory provide positive answers.[11]

The pervasiveness of circularities poses a question that is closely related to that of the existence of fixed points. Namely that of finding ways of "grounding" the circularities. That is, to find an object that can be seen as isomorphic to the "last" object in the chain obtained by unfolding the circular loop. This, of course *seems* impossible if the process

[10] Keynes analyzes the behavior of rational agents in the market through an analogy with a newspaper contest, in which entrants are asked to choose the six most attractive faces from a hundred photographs. Those who pick the most popular faces are eligible for a prize (see Keynes 1936, p. 156).

[11] The solution adopts the form of a *natural transformation*, i.e. a transformation from a functor to another, preserving the inner structure of the corresponding categories. In Vassilakis (1991) it is shown that a fixed-point is the image of a succession of natural transformations, each one applied to a step of the circular process.

involves an *infinite regress*. But this does not preclude the existence of a limit at an infinite ordinal.[12]

These questions are certainly important for the study of *supertasks*,[13] which albeit being only theoretical constructions may work well as idealizations of some real-world processes (Svozil 2003). For instance, situations lurking in close variants of the circular processes discussed above, particularly when they require a transfinite number of steps. In the latter case a process is called a *hypertask* (Clark and Read 1984).

If no solutions can be ensured to obtain at the first infinite ordinal level, the only alternative is to accept that the world in the model is "open" and therefore prone to new and unexpected outside interventions. This is of course the realm of many scenarios studied in theoretical computer science, related to so-called *coinductive* tasks performed by automata (i.e. tasks completed step by step with a stopping condition Rutten 1998). Categorical tools allow to see them as *coalgebras*.[14] This way of viewing transfinite circular processes can be also adopted for the aforementioned applications in economics.

In more conventional terms all the circularity questions amount to find a "solution" to the following "equation",

$$\alpha = \langle s, \mathcal{B}(\alpha) \rangle$$

where s represents the objective elements in the problem, including outside contingencies (due to the open world assumption) while α is obtained by the iteration of a process represented by $\mathcal{B}(\cdot)$.

In an alternative set-theoretical framework[15] an entity like α is admitted. The same happens in more general settings where it can be seen as a (categorical) fixed-point (Moss and Viglizzo 2004).

In economic matters α can be a profile of beliefs, actions, rules, expectations, etc. It acts both as the *result* and the *condition* for decision making. And it can arise from the *aggregate* decisions of the agents, through a $\mathcal{B}(\cdot)$ that can be very complex (involving a large number of steps of computation) or it can be an *emergent* property, i.e. an unexpected consequence of the interactions.

This general and abstract problem captures most of the main issues in theoretical economics, from partial to general equilibrium questions, as well as all the problems involving strategic behavior and asymmetric information. The mathematical tools applied to its study yield different solutions under different settings and parameter structures. But it is clear that only a very general mathematical framework can yield the appropriate responses.

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12 Recall that *ordinal* numbers identify ordered sets. If a sequence is finite, its ordinal equals the number of its elements. If a sequence can be enumerated by the natural numbers, its corresponding ordinal number is called ω, the first infinite ordinal. But other infinite sets can be ordered in distinct ways, yielding other ordinal numbers, called *transfinite* (Bagaria 2014, section 3).

13 "The term 'supertask' [...] designates [...] an infinite number of actions performed in a finite amount of time" (Pérez Laraudogoitia 2013, Introduction).

14 A *coalgebra* is a categorical construction, defined in terms of a functor from a category to itself. If the category is $\mathcal{C}$ and $F : \mathcal{C} \to \mathcal{C}$ a functor, a coalgebra is an object X in $\mathcal{C}$ together with a morphism from X to $F(X)$ (Adámek 2005).

15 One of the Zermelo-Fraenkel axioms of set theory is the axiom of *regularity*, which precludes sets like $a = \{a\}$, which are defined in terms of themselves, called *non-well founded* (Barwise and Moss 1996).

5.2 Data and A Priori Models

Let us now consider how category theory can provide the right tools to connect data with theoretical models. The idea is that any such exercise should have access to a class of *rough data*, which has to be translated into the theoretical language. The relations and objects of study must be clearly distinguished (Tohmé and Crespo 2013).

Suppes' *models of data* provide a set-theoretical way of doing this (Suppes 1962). But a categorical extension allows to enrich this analysis. A theoretical model of the domain, must be established, in such a way that a category-theoretical *adjunction*[16] can be established between it and the structured data model. The idea is that a sort of data/theory duality must be specified (Tohmé et al. 2015).

Finally, and here is where the real gist of the mathematical framework comes at play, in the *topos* in which the potential adjunctions between data and theoretical models operate, an optimal one must be found. Particularly the one with the highest valuation.[17] This means that the "best" theoretical model is obtained, one that is closer to the real world data (Doering and Isham 2011; Náufel do Amaral and Haeusler 2007).

5.3 Discussion

The integration of the aforementioned components in the analysis will yield a overarching view of economic phenomena, in which close relations with data are accompanied by the adequate treatment of the ever present circularities of social life. It is for sure that Category Theory provides the right tools for this task. Particularly important, as we tried to make clear, is the language used in the last decades to unify and make rigorous mathematical thought.

6 Conclusions

Human reason is a weak instrument with a limited capacity for intuition. It proceeds step by step to achieve knowledge. mathematics is a powerful tool enormously useful in assisting human reasoning. It correctly expresses relations and structures and embodies the necessary rules for a correct thinking. Sciences, including economics, benefit a lot from using mathematics. However, not all mathematical developments are adequate for any kind of reality. In the second section of this paper, we have sketchily presented the twentieth century predominant conceptions of mathematics. We consider that Quine's assimilation of these conceptions to medieval accounts of universals is sharp and enlightening. When trying to apply mathematics to real phenomena, conceptions of mathematical entities as abstract real objects (logicism), mind constructions (intuitionism), or empty names (formalism), do not seem to be actually suitable. In the paper we maintained that a twentieth century's neglected conception, namely, mathematical entities as universals present in singular objects, is what is needed. This is the aristotelian approach to mathematics, which adapts to the changing and contextual character of the human realm. After reviewing in Sect. 4 the connection of mathematical economics with its contemporary mathematical

[16] An adjunction between two functors is such that their composition recovers the identity, i.e. making one of the functors kind of the "inverse" of the other (Ellerman 2007).

[17] Given that a topos has a logical structure sufficiently rich to develop 'ordinary mathematics', it comes equipped with a way of ranking the validity of propositions (technically, a *subobject* classifier).

ideas, in Sect. 5 we proposed a new mathematical approaches to economics, based on Category Theory, in accordance with the lastly mentioned definition of universals. By introducing possible applications, we showed how this conception adapts to the economics' subject circularity, context-dependence, and inexactness, calling for a tight relation with empirical data.

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