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Abstract. Macroeconomic crises are events marked by "broken promises" that shatter the expectations that many agents had entertained about their economic prospects and wealth positions. Crises lead to reappraisals of the views of the world upon which agents had based their expectations, plans and decisions, and to a reconsideration of theories and models on the part of analysts. A crisis triggers widespread efforts of abduction in search of new hypothesis and explanations. In this paper we will explore, in particular, the abductions that analysts may apply after a crisis and see how they reveal the prevalence of "wrong" abductions at the onset of the crisis. In order to carry out this exercise, we study the general role of abduction in economic analysis, both theoretical and practical. Economic theory generally proceeds by constructing models, that is, mental schemes based on mental experiments. They are often written in mathematical language but, apart from their formal expression, they use metaphors, analogies and pieces of intuition to motivate their assumptions and to give support to their conclusions. We try to capture all these elements in a formal scheme and apply the ensuing model of abduction to the analysis of macroeconomic crises.

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1 Introduction

The extensive existing literature on the methodology of economics does not mention abduction as a step of economic reasoning nor usually reflect on the origin of economic hypotheses. However, given that abduction is a necessary stage of every scientific development it is also used in Economics. Moreover, in the case of economic phenomena (and more generally, in social contexts) an additional role of abduction has to be taken into account. Abduction, as a type of inference, not only plays a role in social science models building, but also in the behavior of the agents whose actions are being modeled. That is, economists (and other social scientists) have to abduce their models on the basis of agents' abductions. Besides, these agents' abductions are subsequently based on their beliefs about other agents' abductions. For example, in order to predict future events and thus do profitable investments, agents should consider not only their beliefs but specially other people beliefs, the average opinion. The analysts have to abduce hypotheses about behaviors based on abductions of abductions. Keynes exemplifies this situation by a very well-known (by economists) metaphor of newspapers beauty contests. The competitors have to decide, from a large set of pictures, which are the prettiest faces. The prize, however, is awarded to the competitor whose choice is closest to the average decision (which would not necessarily be his actual choice but the choice he suspects the others will do). In this case, Keynes affirms, we reach "a third degree", and he adds that there are some who practice the fourth, fifth and higher degrees [22]: we scientists, abduce what agents abduce that other agents abduce and so on. This has only recently been incorporated into the body of formal modeling tools used in the discipline [5].

While this is well-known among economists, the outbreak of large scale crises usually poses hard questions about the predictive abilities of Economic Theory as well as of the individual agents in real-world economies. It is in such events that abduction plays a central role. Agents may have formed wrong expectations about their future prospects, which leads to a number of promises and compromises that are inevitably broken in a crisis. On the other hand, analysts, seeing this, have to abduce a cogent explanation for the outbreak of the crisis, in which the wrong abductions of the agents are a key component.

The goal of this paper is to analyze the role of abduction in the explanation of the causes of economic crises. In order to achieve this, we present a wider discussion on the nature and role of abductive reasoning in Economics. The plan of the paper is as follows: in Section 2 we will introduce the notion of abduction that we will use in the paper; Section 3 provides a general account on how abduction operates in economics; Section 4 presents a formal framework to show how abduction is performed and how important are certain guiding criteria in the process. Finally, Section 5 presents the core of this paper, the application to the abduction of crises.

2 Abduction and Inference to the Best Explanation

The aim of this Section is to present the notion of Abduction and "Inference to the Best Explanation" (IBE) that we will consider in the paper. We are aware that there are different possible interpretations and classifications of these notions. However, we do not intend to focus on these possible distinctions but to adopt one and to try to show how it applies in Economics. We would roughly affirm that our concept of Abduction is Peircean, as interpreted in Rescher [37], and that our notion of IBE is taken from Niiniluoto [33] and Lipton [29].

Although Aristotle discussed abduction under the name of **apagoge** (in *Posterior Analytics* I, 13), the modern view of abduction was firstly formulated by Charles S. Peirce. Two meanings can be discerned in his use of the term. First, he considers abduction as a type of logical inference. While deduction infers a result from a rule and a case, and induction infers a rule from the case and the result, abduction infers the case from the rule and the result. So understood, abduction can be identified with the fallacy of affirming the consequent. Thus, its result can only be conditionally accepted.

In a second sense, Peirce sees abduction as a way of arriving at scientific hypotheses. He formulates it as [36, 5.189]:

- The surprising fact C is observed.
- But if A were true, C would be a matter of course.
- Hence, there is reason to suspect that A is true.

The conception implied in this second formulation is more general than the first. A might be a case or a hypothetical rule (see [33]). However, Peirce states that, given that the fallacy of affirming the consequent remains, this procedure of discovery and postulation of hypotheses is only a first step of scientific research. That is, abduction in this second sense is a heuristic method assisted by some criteria formulated by Peirce. For him, the hypotheses should be explanatory [36, 5.171, 5.189, 5.197], economical [36, 6.395, 6.529, 8.43] and capable of being tested in experiments [36, 2.96, 2.97, 4.624, 5.597, 5.634, 8.740].

[36, 2.756–760] distinguishes three forms of induction: 1) crude induction, i.e., every day life empirical generalizations; 2) quantitative induction, i.e., statistical induction and, 3) qualitative induction, "the collaborative meshing of abduction and retroduction, of hypothesis conjecture and hypothesis testing" [37, 3]. This abduction corresponds to its second formulation and "retroduction [to] the process of eliminating hypotheses by experiential/experimental testing" [37, *ibid.*].

Aliseda [1] holds that

Abduction is thinking from evidence to explanation, a type of reasoning characteristic of many different situations with incomplete information. Note that the word explanation – which we treat as largely synonymous with abduction – is a noun which denotes either an activity, indicated by its corresponding verb, or the result of that activity. These two uses are closely related [...]. The process of explanation produces explanations as its products [...].

Boer ([4]) explores the meaning of explanation for Peirce concluding that we can find in him elements of today prevailing models of explanation. It is clear that explanation, in usual examples of abducting often points to causes: "we want to know the cause" [36, 7.198; see also 2.204, 2.212, 2.213, 3.395, 3.690, 7.221]. Let us consider this put in [2]:

You observe that a certain type of clouds (nimbostratus) usually precede rainfall. You see those clouds from your window at night. Next morning you see that the lawn is wet. Therefore, you infer a causal connection between the nimbostratus at night, and the lawn being wet.

There is a background knowledge that helps in identifying the explanation, and consequently, the cause¹. The final aim of scientific knowledge according to Peirce is, as Rescher remarks and argues, "the actual truth" $[37]^2$. Boersema, however, contends that Peirce does not take into account only metaphysical (causal) aspects in his account of explanation but also epistemological and axiological elements, in the context of a broad theory of inquiry. The need of retroduction as a second step of Peircean qualitative induction means, however, that the second sense of abduction is a way to suggest hypotheses or possible explanations pointing to true causes, but not a sufficiently justified way to accept them. Nevertheless, Niiniluoto [33] indicates that Pierce considers "an extreme case of abductive inferences" that are "irresistible or compelling" and come to us "like a flash" [36, 5.181]. In these cases, Niiniluoto contends, "for Peirce [abduction] is not only a method of discovery but also a fallible way of *justifying* an explanation" [33, italics in the original]. That is, the strength of this flash would produce a change of the epistemic state of the agent.

Niiniluoto thus distinguishes the procedure of suggesting hypotheses embodying a "weak conception" of abduction and the justification of the hypotheses as a "strong conception" of it. He equates this latter to an Inference to the Best Explanation (IBE): "in the strong interpretation, abduction is not only an inference to a potential explanation but to the *best explanation*" [33, italics in the original]. In short, the weak conception is the best way of arriving at hypotheses, but does not justify them. The strong conception, in turn, is a fallible way of justifying explanations. This latter conception implies a change of the epistemic state of the agent by which she accepts the hypothesis, acquiring new knowledge [44]. Obviously, this acceptance does not mean that the hypothesis is infallible: it is just an accepted hypothesis.

[29] considers IBE as a tool of exploration, generation and justification of the hypotheses. Cresto [9, 10] proposes conceiving IBE as a complex process

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¹ A subsequent problem would be to clarify what is the meaning of cause for Peirce. For example, he criticizes the "grand principle of causation" [36, 6.68]. However, it would go beyond the aim of this paper to deal with this topic.

² Peirce's notion of truth is another topic the paper will not deal with (see [15]).

which proceeds in two steps: the abductive stage and, after testing, the selective stage, in which the epistemic state of the agent changes. Developing Levi's [27, 28] expected epistemic utility theory, she applies this theory to the IBE, considering the epistemic virtues of simplicity (or parsimony), unification power, fertility, testability, economy, and accuracy as essential elements of her proposal. Similarly, Harman [16] proposes simplicity, plausibility and explanation power as criteria for judging the hypotheses while Thagard [44] considers consilience (how much a theory explains), simplicity and analogy. In turn, Lipton [29] mentions unification, elegance and simplicity as virtues leading to what he calls the "loveliest explanation". According to him, this "loveliest explanation" finally becomes the "likeliest explanation". In addition to empirical adequacy, which is required but not sufficient, other epistemic virtues enter into the play in the whole process of IBE. Each context indicates which virtue has more or less weight in the epistemic utility calculus. For example, as Keynes contends, vagueness may be more virtuous than precision when dealing with the complex social realm. For him, elegance and simplicity may be misleading and economy may be a vice instead of a virtue. This is compatible with Peirce's thought: for him "simplicity" does not imply a "simplified" hypothesis, but "the more facile and natural, the one that instinct suggests, that must be preferred" [36, 6.477].

The choice of these criteria is a key point to the process of postulating hypotheses (and, eventually, of justifying them). In Section 4 of the paper (formal framework) we will take into account **simplicity**, **unification power** (external coherence), **internal coherence** and **testability**.

3 Abduction in Economics

We contend that abduction is an essential component of economic analysis, theoretical and practical. Economic theory generally proceeds by constructing models [31], that is, mental schemes based on mental experiments [32]. They are often written in mathematical language but, apart from their formal expression, they use metaphors, analogies and pieces of intuition to motivate their assumptions and to give support to their conclusions (see [13]). In dealing with ongoing economic processes, agents and analysts must generally evaluate whether the situation resembles in a relevant way some instances observed or studied in the past, and whether this warrants applying somehow the "lessons" drawn from those experiences. The problem in judging "whether some pasts are good references for the future" becomes particularly severe when the economy is seen to undergo important changes as in the example that we will provide in Section 6. Simplicity in the Peircean sense, explanatory power, coherence and testability are rather unconsciously considered in this abduction of possible explanatory models.

The retroductive phase also involves problems implying abductive-like decisions. Although it sounds rather obvious, it must be recognized that there is a gap between the formulation of a question to be answered through measurement and the actual measurement providing the right answer. The difference arises from the fact that problems are qualitative while data are quantitative. In consequence, rough data (which certainly are the quantitative counterparts of qualitative concepts) must be organized according to the qualitative structure to be tested. That is, a correspondence between theory and data must be sought. So, for example, in economic theory there exists a crucial distinction between ordinal and cardinal magnitudes in the characterization of preferences. But once measurements are involved it is clear that the theoretical relational structure must be assumed to be homomorphic to a numerical structure [24]. This implies that if there exists a data base of numerical observations about the behavior of a phenomenon or a system, we might want to infer the properties of the qualitative relational structure to which the numerical structure is homomorphic. Of course, this is impaired by many factors:

- The syntactic representation of the qualitative structure can be somewhat ambiguous [3].
- Although the observations fall in a numerical scale, the real world is too noisy, allowing only a statistical approximation.
- The complexity of the phenomena may be exceedingly high. Then, only rough approximations may make sense.

These factors, which preclude a clear cut characterization of the observations, leave ample room for arbitrary differences. In this sense, the intuition and experience of the economist and the econometrician determine the limits of arbitrariness in an abductive-like fashion. As an example, consider the question "Did a specific economy grow in the last year?" To provide an answer, first, one has to define clearly what does it mean that an economy grows and which variables can be used to measure the phenomenon of growth. Economic theory states that economic growth means growth of the national income. But in order to answer the question an economist has to define what real world data will represent national income; i.e. she has to embed the available data into the framework given by the theory. In this case the national product is an available variable which is easy to measure and is considered (theoretically) equivalent to the national income. Therefore it is easy to check out whether the economy grew or not. But in the case where the question is something like "Did welfare increase in the last twenty years?" the procedure is far less simple. How do we define welfare and moreover, how do we make the concept operational? This is where the intuition of the economist is called in. Although theoretical concepts may be lacking, a set of alternative models of the notion of welfare and its evolution in time should be provided in order to check out which one fits better the real world data. When this question is settled it is possible to consider the development of a theory formalizing the properties satisfied in the chosen model. That is, when the abducting process is completed the theory-building phase can start.

The inferences that allow economists and econometricians to detect patterns in reams of data cannot be called statistical inductions. They are more a result of a detective-like approach to scarce and unorganized information, where the goal is to get clues out of unorganized data bases of observations and to disclose hidden explanations that make them meaningful. In other words: it is a matter of making guesses, which later can be put in a deductive framework and tested by statistical procedures. So far, it seems that it is just an "artistic" feat, which can only be performed by experts³.

Let us give a sketchy description of the reasoning process in Economics. Economists have a background of general rules. When a surprising fact appears the first step is to try to come up with an explanation according to those rules. The best explanation obtains by delimiting the possible hypotheses until only one of them remains. In this process the economist uses information about similar situations as well as the features of the specific case to capture simple and coherent hypotheses and models.

We may distinguish the following steps in this process:

- 1. An abnormal event is detected, requiring an explanation.
- 2. The event is carefully described.
- 3. Some stylized facts are extracted from the description.
- 4. Situations sharing the same stylized facts are given particular attention.
- 5. Formal expressions, capturing the relations deemed essential in the explanation of the relevant stylized facts, are formulated.
- 6. Only those combinations of deductive chains and inductive plausibility that are both externally and internally coherent are chosen, discarding other possibilities.
- 7. This provides an original coherent explanation of the event.
- 8. The conclusions are tested.

Abduction is hidden in the whole process, but steps 5 and 6 are mostly deductive. On the other hand, step 8 is also inductive and retroductive. The whole process is a Peircean qualitative inductive process. Good economists have a guess instinct [36, 6.476–477] present in their scientific processes. This is not a mysterious miracle but an intellectual intuition, stemming from a theoretical framework or background knowledge, of experience, of hard work with theories, models and data. This leads good economists to foresee a set of probably successful models. Combining this gift with hard empirical work economists often overcome the problems of under-determination of theories by formulating local or context-dependent theories. Context-dependence is a characteristic feature of IBE [10, 11]. However, the economists always try to improve their models. This is because, given the fluctuating ontological condition of the economic material, a close relation with real situations is needed. The analogies sometimes work and sometimes not. Old or conventional theories may be misleading. Thus, economists need that special "gift for using

³ This might be a reason for why formal logicians, until recently, did not intensively study abduction in contrast to the other forms of inference.

vigilant observation to choose good models" [21, p. 297]. This improvement, however, has a limit. On the one hand, the frequent urgency of decisions that cannot wait for further investigation, and the economy of research ([37, p. 65 ff.], extensively quoting Peirce), actually lead to accept the conclusions as fallible though reasonable inferences to the best explanation. On the other hand, the mentioned problems of quantification - conceptual, institutional, accuracy of data, calculation and even presentation -, also incline to accept a sufficiently probed fallible conclusion as a good one. Given this informal characterization we are interested in providing a formal framework for abduction in the next Section.

4 A Formal Framework for Abduction

What clearly separates abduction and IBE from statistical induction is that they require a previous meta-theoretical commitment. The study of the historical example of how Kepler derived the laws of planetary motion, allowed Peirce to clarify this point. Moreover, Peirce draw from this example some prescriptions on how to perform an abduction. First of all, data (or more generally information) had to be structured by means of Peirce's own classifications of signs [30]. Since in this view every set of data constitutes a sign, it can be classified according to Peirce's exhaustive taxonomy. The advantage of this approach is that there exists only a finite set of possibilities to match with the real world information. Once one of the possibilities becomes chosen, it is assumed to provide a clear statement of the kind of structure hidden in the data, although not necessarily as complex as a functional form.

Peirce's approach can be used as a heuristic guide for the formalization of abduction in economic analysis. This is because the pieces of information available to an economist cannot be all put in the same level. In fact, to classify a set of data in terms of the meaningfulness of the information conveyed is very useful in order to construct a testable hypothesis. While this is a hard task, the remaining chore is still harder: to work on the classified data base, trying to fit it to one of a bundle of possible functional forms.

We will try to make this discussion a bit more formal and develop an approach to qualitative model building in economics. In the first place, we should note that the meaning of **model** in this field is not the same as in mathematical logic. We will try to keep the meaning of the word as used in first-order logic, so in order to explain how abduction helps in economic model building, we need some previous definitions⁴:

Definition 1. Given a first order language \mathcal{L} a structure is $\Delta = \langle \mathbf{N}, \gamma, \mathcal{F}, \Pi \rangle$, where \mathbf{N} is a set of individuals; γ is a function that assigns an individual to each constant of \mathcal{L} , \mathcal{F} is a family of endomorphic functions on \mathbf{N} , while Π is a set of predicates on \mathbf{N} . An interpretation of any consistent set of well formed

 $^{^4}$ For a precise characterization of these notions see [38].

formulas of \mathcal{L} , $\mathcal{T}(\mathcal{L})$ obtains through a correspondence of constants, function symbols and predicate symbols to Δ . A model of $\mathcal{T}(\mathcal{L})$ is an interpretation where every interpreted formula is true.

A structure can be thought of as a database plus the relations and functions that are, implicit or explicitly, true in it. An interpretation is a structure associated to a certain set of well-formed formulas (when deductively closed this set is called a *theory*). If, when replacing the constants by elements in the interpretation and the propositional functions by relations in the structure, all the formulas are made true in the interpretation, all the formulas become true in the interpretation, this structure is called a model. To say that abduction helps in model building means that it is a process that embeds the real-world information in a certain structure that is assumed to be the model of a theory or at least of a coherent part of one.

In economics it is usual to find that there is not a clear distinction between what is meant by "theory" and by "model". One reason is that for most applications, it is excessive to demand a theory to be deductively closed, which means that all its consequences should be immediately available. In the usual practice, statements are far from being deduced in a single stroke. On the other hand -and this explains clearly the confusion between theory and model- most scientific theories have an intended meaning more or less clear in its statements. This does not preclude the formulation of general and abstract theories, but their confrontation with data are always mediated by an intended model [41].

Another concern that may arise from our approach is whether any economically meaningful assertion can be embedded in a first-order language. The point is that most theories of sets, Zermelo-Frenkel and others, intended to provide a comprehensive foundation for mathematics, are first order [12]. Since most of the economic statement can be expressed as set-theoretic expressions, it seems that the previous definition of a structure is enough for our purposes. What we need is to translate the data base of observations into a formal structure such that [26]:

- Each element of interest in the data has a symbolic representation.
- For each (simple) relationship in the data, there must be a connection among the elements in the representation.
- There exist one-to-one correspondences between relationships and connections, and between elements in the data and in the representation.

This representation of the real world information, Λ , facilitates the abduction, by means of its comparison with alternative structures. The result of the abduction will determine an implicit representation of data, as we see if we consider the following definition:

Definition 2. Given a set of structures $\{\Delta_i^{\mathcal{C}}\}_{i \in I}$ where I is a set of indexes, selected for satisfying a set of criteria \mathcal{C} , an abduction is the choice of one of them, say Δ^* , by comparison with Λ .

In words, given a class of criteria, there might exist several (although we assume only a finite number) possible structures that may explain the data in Λ . To *abduce* Λ , is to choose one of them. We have to explain, on one hand, what those criteria might be and, on the other, how a single structure may be selected. With respect to the criteria, notice that in the case of Kepler's abduction he had at least one criterion in mind: trajectories of celestial bodies should be described by simple geometrical expressions. Under this criterion, Kepler had to choose one among a few structures comparing the movements implied by them with the behavior of a given set of real-world elements (the known planets of the solar system). Each of those structures was a simple geometric representation of the solar system. He finally chose the one that fitted the data best.

In general, the criteria represent all the elements that the scientist wants to find incorporated into the chosen structure. Given the criteria in C the set of structures that satisfies them is defined as follows:

Definition 3. A criterion c_j defines a set of structures in which it is satisfied, $\{\Delta_i\}_{i \in I_j}$ (where I_j is a set of indexes corresponding to this criterion). Then, $C = \{c_j\}_{j \in J}$ defines a set of structures $\{\Delta_i^C\}_{i \in I} = \bigcap_{j \in J} \{\Delta_i\}_{i \in I_j}$.

In general, the number of criteria is reduced in order to ensure that the set of possible structures is not empty. The comparison of the structures with the data determines an order on $\{\Delta_i^{\mathcal{C}}\}_{i \in I}$:

Definition 4. Given Λ , and two possible structures Δ_j , Δ_l we say that $\Delta_j \preceq \Delta_l$ if and only if $\mathbf{WFF}(\Delta_j) \overline{\cap} \Lambda \subseteq \mathbf{WFF}(\Delta_l) \overline{\cap} \Lambda$, where $\mathbf{WFF}(\cdot)$ is the set of well-formed formulas corresponding to a given structure and $\overline{\cap}$ is a satisfaction operator.

To complete this definition, we have to provide a characterization of the satisfaction operator $\overline{\cap}$. Notice that if we had used only the set-theoretic intersection \cap we would have missed the point of comparing Λ with the potential structures. Since Λ may just consist of a data base of numerical observations, a qualitative structure may not yield even a single one of those observations and still be meaningful. In order to address this question, we have to consider each relation \mathcal{R} implicit in Λ . Then consider the collection of sets of observations in Λ , denoted 2^{Λ} . Then, an application of the Axiom of Choice for finite sets yields that:

Definition 5. A proposition $\lambda_{\mathcal{R}}$ satisfies Λ if and only if for every finite subfamily sets in Λ there exists a choice S such that for every $a_1, \ldots, a_n \in S \subseteq 2^{\Lambda}, \mathcal{R}(a_1, \ldots, a_n)$.⁵

Consider then, the family of the propositions $\lambda_{\mathcal{R}}$ for all relations \mathcal{R} defined over Λ . These relations may represent the closeness of numerical values, or

⁵ This follows in a logic defined over a hypergraph in which the observations constitute the nodes and sets of observations under the relation \mathcal{R} the hyperedges [23].

the fact that they belong to a given interval or, closer to Peirce's aim, a hierarchy of observations, ones deemed more relevant than the others. In any case each of these formulas abstract away from the data base. But then:

Definition 6. Given a structure Δ , $\mathbf{WFF}(\Delta) \cap \Lambda = \{\lambda_{\mathcal{R}} : \Delta \models \lambda_{\mathcal{R}}\}$, where \models is the classical relation of semantical consequence.

That is, $\mathbf{WFF}(\Delta) \overline{\cap} A$ consists of those $\lambda_{\mathcal{R}}$ that are satisfied by Δ , and can be seen as well-formed formulas shared by the data base and the theory for which Δ is a model. Finally, the relation among structures \preceq simply yields for every pair of structures, Δ_l, Δ_j a preference for the structure, say Δ_l , that satisfies not only the same formulas of the data base as Δ_j but also some more. Notice that the class of the wffs $\lambda_{\mathcal{R}}$ determine, as much as the candidate structures, the resulting order \preceq .

Even if this description is sound, in practice there exist serious difficulties associated with the detection of patterns and relations in a numerical database. This fact is well known by statisticians:⁶ an approximate generalization is, according to any statistical test, indistinguishable from the form of a wrong generalization. Even if statistical inferences may preclude hasty generalizations, the fact is that qualitative data may not correspond directly to quantitative forms that can be statistically supported.

Other (non-statistical) methods lead to similar problems. Computational intelligence only provides rough approximations to the task of theory or model building. Systems like BACON (in any of its numerous incarnations) despite their claimed successes are only able to provide *phenomenological* laws [40]. That is, they are unable to do more than yield generalizations that involve only observable variables and constants. No deeper explanations can be expected to ensue from their use.

In the process of inquiry carried out by economists, the human side has a crucial task, not yet fully elucidated in the literature: the formation of concepts and the elicitation of qualitative relations. In fact, experts excel in detecting patterns and relations in disordered and noisy data. Of course, as it is well known in Combinatorics, more precisely in Ramsey Theory [14], with enough elements a regular pattern will exists, be it meaningful or not. In any case, an expert uses the patterns and relations he finds or imposes over the database and this is represented above by the procedure of selection S.

Based on this possibility of finding expressions that "refine" the crude information in Λ we have the following result:

Proposition 1. There exists a maximal structure Δ^* in the set $\{\Delta_i^{\mathcal{C}}\}_{i \in I}$ ordered under \leq .

Proof. First of all we will show that \leq is a partial order, i.e. that it verifies the following properties:

⁶ See [39].

- **Reflexivity**: since $\mathbf{WFF}(\Delta_i) \overline{\cap} \Delta \subseteq \mathbf{WFF}(\Delta_i) \overline{\cap} \Delta$ then $\Delta_i \preceq \Delta_i$.
- Transitivity: if $\Delta_j \preceq \Delta_l$ and $\Delta_l \preceq \Delta_k$ then $\mathbf{WFF}(\Delta_j) \overline{\cap} \Delta \subseteq \mathbf{WFF}(\Delta_l) \overline{\cap} \Delta$ and $\mathbf{WFF}(\Delta_l) \overline{\cap} \Delta \subseteq \mathbf{WFF}(\Delta_k) \overline{\cap} \Delta$. By transitivity of \subseteq it follows that $\mathbf{WFF}(\Delta_j) \overline{\cap} \Delta \subseteq \mathbf{WFF}(\Delta_k) \overline{\cap} \Delta$ i.e. that $\Delta_j \preceq \Delta_k$.

Since we assume that Δ is finite, $\langle \{\Delta_i^{\mathcal{C}}\}_{i \in I}, \preceq \rangle$ is bounded: for any Δ_i such that $\Delta \subseteq \mathbf{WFF}(\Delta_i)$ there is no other Δ_l such that $\Delta_i \prec \Delta_l$. Therefore, using Zorn's Lemma it follows that $\langle \{\Delta_i^{\mathcal{C}}\}_{i \in I}, \preceq \rangle$ has a maximal element. \Box

A trivial case of a maximal structure Δ^* arises when $\Delta^* \models \Lambda$. That is, when all the observations in the data base are satisfied in the structure. But, as said, this is not only difficult to be found, but also undesirable, if the data base includes noisy and otherwise imprecise observations.

So far, many structures may be chosen. Sufficient conditions for uniqueness can be achieved if certain criteria are included in C:

- **Definition 7.** \mathbf{c}^{min} (Minimality): given two structures Δ_i, Δ_j , such that $\mathbf{WFF}(\Delta_i) \subseteq \mathbf{WFF}(\Delta_j)$ and $\mathbf{WFF}(\Delta_j) \not\subset \mathbf{WFF}(\Delta_i)$, select Δ_i .
- \mathbf{c}^{comp} (Completeness w.r.t. Λ): given two structures Δ_i, Δ_j , where $\Lambda \subseteq \mathbf{WFF}(\Delta_i)$ but $\Lambda \not\subset \mathbf{WFF}(\Delta_j)$, select Δ_i .
- \mathbf{c}^{conc} (concordance w.r.t. Λ): a given structure Δ is selected if for every $\lambda_{\mathcal{R}}$ derived from Λ , either $\lambda_{\mathcal{R}}$ or $\neg \lambda_{\mathcal{R}}$ belongs to $\mathbf{WFF}(\Delta)$.

Then we have the following result:

Proposition 2. If $\{\mathbf{c}^{min}, \mathbf{c}^{com}\} \subseteq C$ and the set of possible structures is otherwise unrestricted, Δ^* is unique.

Proof. There are two cases to consider. If $\{\Delta_i^{\mathcal{C}}\}_{i \in I} = \emptyset$ then we can define $\Delta^* = \emptyset$, which is trivially unique. If $\{\Delta_i^{\mathcal{C}}\}_{i \in I} \neq \emptyset$, according to \mathbf{c}^{comp} , $\Delta \subseteq \mathbf{WFF}(\Delta_i^{\mathcal{C}})$ for all $i \in I$. On the other hand, according to \mathbf{c}^{min} there is a minimal $\Delta_i^{\mathcal{C}}$ such that $\Delta \subseteq WFF(\Delta_i^{\mathcal{C}})$. Since one of the possible structures is a Δ' such that $\Lambda = \mathbf{WFF}(\Delta')$ we can define $\Delta^* \equiv \Delta'$, which is unique. \Box

Similarly:

Proposition 3. If $\{\mathbf{c}^{min}, \mathbf{c}^{conc}\} \subseteq C$ and the set of possible structures is unrestricted, Δ^* is unique.

These results shows that a unique structure can be selected if the restrictions on possible structures obey to methodological criteria like minimality, completeness or concordance. This is not without a cost: if the only wffs in the chosen structure are the ones drawn from the database it is not possible to provide more than a description (data fitting) of the available information. This means in turn that if only methodological criteria are to be used, the result of the inference is the generation of a prototype, i.e. only a statistical inference is performed. In Economics these criteria are usually violated since sometimes inferences are drawn from partial samples from a bigger database

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(violation of \mathbf{c}^{comp}), some observations are discarded as outliers (violation of \mathbf{c}^{min}), or some information in the database is not used (violation of \mathbf{c}^{conc}). Nevertheless they represent extreme case of very desirable properties: minimality involves **simplicity** while completeness and concordance approximate **unification power** (i.e. external coherence). On the other hand, the fact that the abduction yields a structure implies **internal coherence**. A final requirement, **testability**, is satisfied when the structure yields observable outcomes not found in Λ , that have to be checked out in the real world.

The chosen relational structure Δ^* is, as said, conceived as a model of a theory \mathcal{T} , which in economic parlance is the actual "model" sought for. To derive this \mathcal{T} , one might choose one from a collection of closed sets of wffs of \mathcal{L} , each one having Δ^* as a model. One candidate is just $\mathbf{WFF}(\Delta^*)$ itself as a bundle of first-order formulas. Other possible theories may involve information that is certainly not present in the data. In the case that $\mathcal{T} \equiv \mathbf{WFF}(\Delta^*)$, the theory is, as said, called *phenomenological*. Otherwise, the theory is said *representational* and involves to postulate non-observable properties and entities.

As seen above, the burden of the task of performing abductions is on the set of criteria used. Although this is true for every science, in human affairs it seems that the hidden assumptions account for a good deal of surprising results that lead, in turn, to policies affecting the lives of the members of entire societies. This is, in fact, the case of economic crises.

5 Abduction and Macroeconomic Crises

The meaning of the term crisis in the economic literature is not without ambiguity. However, there is a commonsense group of characteristics that we will retain here as defining a set of critical macroeconomic events: (i) they have a large scale, in the sense that they are reflected in wide swings in macroeconomic aggregates and affect the behavior and economic performance of a population as a whole, (ii) they are perceived by most agents as a sizeable disturbance in their economic life and prospects, (iii) they typically involve moments of abrupt economic change marked, for example by shifts in asset prices much sharper than normal measures of variability, or by "big news" like the failure of some large firm and (iv) they are memorable events, which lead many people (agents and perhaps analysts) to reconsider plans and beliefs or, at least, to engage in active after-the-fact learning in order to revise opinions and expectations. Economic depressions, financial crashes, hyperinflations and collapses of monetary and exchange regimes would belong to this category, although clearly they form a heterogeneous set in terms of the economic processes at work. We shall concentrate here on phenomena of the family that includes the ongoing worldwide crisis and the Great Depression (lumping together those events implies already a presumption of comparability despite their clear dissimilarities, but it may be admissible at this point

because of the widespread use of the analogy and the related search for similarities and contrasts).

Concern for the study and the understanding of crises is actually older than macroeconomics as an established discipline and it has operated historically as a strong motivation to investigate in the field. Modern macroeconomic theory, on its side, has increasingly become committed to a set of analytical and procedural presumptions, which lead to look for representations of macroeconomic behavior as the result of well coordinated (except for some noise which acts as an additional constraint) optimal decisions of agents, equipped with rational expectations, that is with knowledge of the probability distributions relevant for their plans. These research criteria, sometimes elevated to the rank of methodological prescriptions, can be seen as the outcome of past debates on the theory of macroeconomic fluctuations and inflation, which generated dissatisfaction with earlier theories. At the same time, their application to the study of crises, as if they could claim a universal range of validity, has been subject to paradoxes and problems in the interpretation of salient facts, which seem to call for new searches of the abductive type.

Consider the following facts, which describe the onset of a crisis:

- 1. A large, relatively closed economy (or, to make the point more starkly, the world economy as a whole) undergoes a period of rapid expansion, marked by substantial technical changes and the emergence of new patterns of the division of labor, together with strong increases in aggregate demand.
- 2. The volume of credit rises strongly, through the issue of wide variety of instruments. Both consumers and firms show willingness to enter into debt, and higher asset prices indicate the strength of the demand by prospective lenders.
- 3. There are public discussions about the possibility that the economy has grown an unsustainable bubble, and that policy corrections may have to be applied. The authorities decline to do so, with the view that it is not clear that asset prices and credit flows are out of line and, if that was the case, policies can handle a potential correction without much disruption.
- 4. Eventually, doubts about the sustainability of asset prices induce some falls. Banks start showing worsening results due to increasing defaults on their loans. Evidence of those problems reduces asset demands and leads to cautious attitudes by households and firms in their demand for consumption and investment.
- 5. At some point, information emerges about a large mass of bad debts, which threatens banks with failure. The central bank provides massive assistance to troubled institutions. However, credit flows are disrupted, and asset prices fall precipitously.
- 6. The economy enters into a sharp recession, with substantial falls both in investment and consumption, and higher unemployment. Bad news about the real economy are followed by further declines in the demand for private bonds and equities. At the same time, interests on the public

debt drop to very low levels. The government decides substantial increases in its spending, and runs large deficits.

This scenario, highly simplified as it is, has still too much detail to lend itself to a precise analysis. Consider then, with the facts just described as background, the question of what would explain the wide swings in macroeconomic activity and in asset valuations that is, of what set of features of the economic process are crucial in determining those fluctuations, and thus deserve a closer exploration in view of advancing towards a general account of crises. This question requires some initial definition of a framework of analysis, and a narrowing down of the set of alternatives to be contemplated. Among the various possibilities we will contemplate the following arguments that could be used to rationalize the choice of the different hypotheses as basic elements of the approach to the subject:

- Δ₁: The statistical properties of macroeconomic aggregates (output, consumption, investment, employment) resulting from historical data can be represented to a reasonable degree of approximation by DSGE's (Dynamic Stochastic General Equilibrium Models) where the behavior of agents is described through the solution of dynamic programming problems in stochastic environments where the main impulses driving the system are random, exogenous, shocks to the aggregate productivity of the economy, or shifts in monetary policy which distort labor supply and demand decisions, and where the expectations of the agents are rational. The analysis should be based on such a construction, with the presumption that the severity of the macroeconomic swings would be determined by the extraordinary magnitude of the shocks hitting the system⁷.
- Δ₂: The distinguishing feature of the episodes under consideration is the amplitude of the fluctuations in credit, from a phase of easy financing which promotes exaggerated valuations of assets and unsustainable buildups of credits, to a collapse of the bubble where the supply of lending falls dramatically. Monetary policy is the main regulator of credit conditions. Therefore, the crisis can be traced to an unduly lax attitude of monetary authorities, and a later tightening, associated with a strong credit contraction [43, 34, 35].
- Δ₃: All crises of this type are ultimately great swindles, where some groups of economic agents gain to the detriment of others. Thus, a crisis reveals deep problems of incentives, like rewarding bank officers for making loans that they know will not be repaid, or giving bonuses to executives for short run profits when their actions will eventually lead to bankruptcy. In extreme cases, like the pyramid schemes posing as investment funds, even seemingly sophisticated traders were cheated. The behavior of trying to take advantage of other persons was also a decisive factor in asset valuation, as rational agents kept demanding stocks and bonds, knowingly

⁷ This line of explanation has generated a significant literature on past episodes of economic depression [8, 20]. For the crisis started in 2008, see [6] and [7].

"riding the bubble" because they understood that they would be able to sell at prices before the collapse [17, 19].

 Δ_4 : The marking characteristic of crises like the one being analyzed is • the widespread frustration of economic plans and expectations, reflected particularly in large- scale defaults on financial commitments. Such crises involve as their crucial element a drastic fall in the wealth perceptions of substantial groups of agents, implying that previous decisions on production, spending and credit supply or demand had been based on wrong anticipations. Crises are memorable events, which lead to revisions of beliefs on the part of economic agents, policymakers and analysts: the search for lessons to learn from such episodes indicates that models of analysis and decision are in fact being reconsidered. Moreover, crises tend to occur after periods where changes in the present or prospective configuration of the economies (due to technological developments, policy reforms or a modified international environment) are apt to sustain expectations of future increases in real revenues and thus to promote a perception that a credit boom has fundamental underpinnings. Those swings in beliefs and expectations in economies undergoing changes in structure or performance should be one of the basic objects of analysis research and a necessary element in macroeconomic models seeking ranges of validity that include those episodes [25, 18, 42].

The previous arguments are not necessarily incompatible (one can for example insist in the central role of changes in expectations without denying the relevance of monetary policies or incentive effects in the actual course of events), but they can base different approaches to the subject, and also generate quite distinct policy implications. We shall maintain here that Δ_1 is actually the most general, in that it points to an essential component of the phenomena. The alternatives, as those briefly outlined above, would not qualify as cores of a theory of crises. Nevertheless, they qualify as explanations for a crisis of the types that have been usual in the world since 1994⁸.

To abduce an explanation we need, as said in the previous section, a family of criteria. The following are adequate in this case:

- **c**^{*A*}: The hypotheses should satisfy the onset of the crisis (1 to 6, above).
- \mathbf{c}^B : The explanations should be internally consistent.
- \mathbf{c}^C : The border conditions of each hypothesis should be observable.

The cogency of these criteria is evident. \mathbf{c}^A just captures the idea that, if we seek an explanation for a crisis, it should describe its process. On the other hand \mathbf{c}^B indicates that inconsistent structures should be disregarded. Finally, \mathbf{c}^C , indicates that any reference to external variables or conditions invoked in an explanation should be testable (otherwise, the explanation would hinge on unverifiable assumptions).

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⁸ That is, the Mexican, South-Asian, Russian, Brazilian, Turkish, Argentinian, Dot.com, and the Subprime crises.

To proceed, let us note that in principle, all four hypotheses satisfy \mathbf{c}^A . That is, a crisis that follows each of those prescriptions will undergo steps 1 to 6. But the DSGE models with rational expectations (Δ_1), even when augmented with financial propagation mechanisms beg the question about the impulse that would shock the system and about the ex-ante probability that agents may have assigned to a large disruption of their plans. That is, Δ_1 may fail to satisfy $\mathbf{c}^{C,9}$

As for the oscillations in monetary policies (Δ_2) , it is true that very low interest rates would raise asset prices transitorily and also cause transitory increases in aggregate spending. However, if economic actors anticipate correctly an interest rate fluctuation and its consequences, it will not induce defaults or disturb the wealth perceptions of agents. A similar argument holds for malineentives and fraud (Δ_3). In particular, a too risky loan policy by bank managers, if perceived as such by the public, would lead to falls in the market valuation of those institution and, in the limit, to a refusal to buy the debt or the equity of those institutions before the bubble develops; the expected possibility of moral hazard through government bailouts would either draw a response of policy-makers or, if these are not willing to react, and that is well understood by the private sector, it would provoke a precautionary response of taxpayers and holders of government bonds who discount the coming fiscal burden. In sum, there seems to be no adequate alternative to establishing a close association between expectational errors and swings in asset prices and credit flows marked at some point by uncommonly large defaults and by sharp and widespread cuts in consumption. That is, Δ_2 and Δ_3 do not satisfy \mathbf{c}^B .

In summary, only Δ_1 and Δ_4 might satisfy the three criteria for abduction. Criteria like \mathbf{c}^{min} or \mathbf{c}^{com} are not strictly satisfied by either candidate, so the only remaining possibility is to establish an order \leq between Δ_1 and Δ_4 . In this sense, we have that $\Delta_1 \leq \Delta_4$, since Δ_4 may yield an explanation for a crisis even in the absence of external shocks.

The previous argument restricts the classes of theories among which the search for a representation of crisis-type events would proceed to those for which Δ_4 might be part of their models. It may be noted in this regard that some knowledge about the probability of behaviors leading to crises would be of great importance. In any case, an essential part of an applicable model would be a specification of how agents form expectations about their future incomes and about the returns of various assets, in economies with evolving configurations. In one way or another, this would require addressing concretely how people determine their representations of economic conditions to come and, consequently, how they understand their environment and project its trends.

 Δ_4 raises some important questions in this sense. The most important arises from the realization that the behavior of the agents starts from

⁹ [6] claims that the large increases in the price of commodities during the 2000's act as impulses for the financial crisis started in 2008.

ex-ante abductions on the overall prospects of the economy that end up being obviously mistaken. Why does this happen? It might be because they do not use the right criteria or because they disregard important data. Thus, in fact, the analyst would be involved in a "second order" abduction when trying to understand expectation formation. This certainly seems a difficult matter for inquiry, but one that cannot be left aside when trying to develop workable theories of phenomena of the social importance of macroeconomic crises.

6 Conclusions

The example presented in the last section shows that meta-theoretic assumptions may lead in an otherwise innocent looking abduction to a result that may affect not only our understanding of the behavior of economies but also the actual behavior through the application of economic policies. In this matters it seems that abductive reasoning must abandon the realm of implicit activity to become an open activity, that may be discussed with the same seriousness as the values of statistical estimates.

An abductive inference should be reported providing:

- The set of criteria to be considered, precisely stated.
- The alternative hypotheses that are postulated (obeying to the criteria). Each should be represented by a system of relations, which constitutes a necessary condition for the respective hypothesis.
- The tests showing which of the hypothesis is accepted. The acceptation criteria should be already stated in the set of general criteria.

Therefore, any discussion on the inference can be based either on the criteria used or on the set of postulated hypotheses. In the first case, the criteria may be wrong, biased, insufficient, etc. In the second case, any new hypotheses added to the list may conform to the originally stated criteria. Both types of discussion may enliven the scientific evaluation of the available information.

This analysis, as it has been revealed in the case of macroeconomic crises, must also cover the abductions performed by the economic agents. The results of this line of research may have important practical applications, starting with the design of more effective policies and regulations on processes in which uncertainty is pervasive.

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